Performance Based Learning and Assessment Task

How to Calculate and Analyze the Equation for a Parabolic Path

I. **ASSESSSMENT TASK OVERVIEW & PURPOSE:**
Students will apply their knowledge of the curve of best fit and quadratics to create numerous parabolic paths from ordered pairs and find the respective quadratic equations.

II. **UNIT AUTHOR:**
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III. **COURSE:**
Algebra II

IV. **CONTENT STRAND:**
Algebra II – AII.9

V. **OBJECTIVES:**
Students will use their knowledge of a coordinate plane, finding the curve of best fit, and the properties of quadratics to: 1)Choose three points that generally form a parabola, 2)Use that data to discover the quadratic equation for the curve of best fit, 3)Calculate the vertex and one other additional point, 4)Analyze and answer various questions concerning the respective quadratic.

VI. **REFERENCE/RESOURCE MATERIALS:**
Students will use an algebra II textbook (provided in the classroom), graphing calculator (TI), writing utensils, computer with ActivInspire (or graphing paper and a straight edge), loose leaf paper/printed computer paper, handout with directions (provided), and colored pencils (if a computer with ActivInspire is not available).

VII. **PRIMARY ASSESSMENT STRATEGIES:**
Students will be assessed on: 1)How accurate their equations are for the respective data points, 2) The accuracy of their calculations, 3)Showing necessary, 4)Neatness of their math work parabolic paths, 5)Clarity and creativeness in illustrating their parabolic paths

VIII. **EVALUATION CRITERIA:**
Self-assessments/ teacher rubrics are attached at the end of this document, as well as a product of what is expected of the students and will act as a benchmark for their achievement.

IX. **INSTRUCTIONAL TIME:**
Two ninety-minute class periods

How to Calculate and Analyze the Equation for a Parabolic Path

Strand
Algebra II

Mathematical Objective(s)
Students will: 1)Calculate the quadratic model/equation for each path based on the three points, 2)Find the respective vertex of that particular parabola, 3)Calculate a 5th point by using the respective quadratic equation, 3)Apply the knowledge of quadratic functions to analyze the parabolic motion

Related SOL
AII.9: Statistics ("The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions")

Materials/ Resources
Students will use: 1)Textbook, 2)Computer with the program ActivInspire or graph paper, straight edge, and colored pencils, 3)Graphing calculator, 4)Loose leaf paper, 5)Flipchart on ActivInspire

Assumption of Prior Knowledge
Students should have a basic understanding of the shape of a quadratic function and the basic concept of the properties of quadratics. The teacher will encourage students to use words such as vertex, standard form, reflected, parabola, quadratic model, zeros/solutions/roots/x-intercepts

Introduction: Setting Up the Mathematical Task

The teacher will ask the students if they have ever thrown an object such as a rock, football, baseball, or played a game such as Angry Birds. The teacher will allow students to think and then pick students to share their experiences with such activities.
The teacher will explain that these are all examples of parabolic paths and that an equation can be found to represent such situations. The instructor will then connect how that equation can then be used to determine various other types of information about that path, such as its highest point, or how long it would take before the object hits the ground.

The teacher will explain to the class that student groups will create three parabolic paths of their choosing and use those general paths to find a quadratic equation for the respective path. Students will use that model to calculate other information such as the vertex, points on the path, and roots. The teacher will project an expectation of what a path might look like for the class.

Students will be placed into groups of 2 by the teacher to ensure positive working environments. Each group will be given the needed materials and calculator steps in order to complete the project. In addition, each group will get a "Quadratic Analysis" sheet to be completed.

Students will be told by the teacher that their general paths and all calculations must be completed in one 90 minute class and that all work must be shown on a "Calculations Sheet." The students will then use the next class period to complete the creative portion of displaying their work within ActivInspire. In addition, during the 2nd class period, each group will complete a "Self-Assessment" based on their project, along with completing the "Quadratic Analysis."

Student Exploration

Small Group Work

Formulating General Paths and Calculating Equations and Other Key Points- (1st 90 minute class period):
The teacher will hand out the Self-Assessment Rubric and the steps to follow in order to find the three quadratic equations for each path, along with the other key calculations. ActivInspire will not be used for the calculations in this portion.
Students will follow the steps in handout. The teacher should also remind students about rounding to the correct amount of digits to ensure a precise result.
A whole class discussion should take place the last 5 minutes of class. Teacher should ask, "What was or were the most difficult part(s) of the day?" and remind students of what they will be doing the next class period.

Creative Design of the Parabolic Paths- (2nd 90 minute class period):
The teacher will remind each group to look at their self-assessment. The teacher also explains that groups can copy and paste images from the internet into their flipcharts if cited properly. Students will then decorate their flipcharts. Once their flipcharts are completed to their satisfaction, they will go to "edit" then "grid" and uncheck the "visible" box so that their project is more appealing to look at.

The teacher will encourage students to use key terms such as, radius, parallel, perpendicular, semi- circle, reflection, and symmetry.

Quadratic Analysis- (2nd 90 minute class period):

Groups will use their flipcharts and "Calculations Sheet" in order to answer the questions on the "Quadratic Analysis." Students will also have the textbook and calculators to use in order to complete their analysis.

It is important that the teacher will continuously walk around the room and stressing the point to make responses to the questions clear and thorough. The teacher should remind the students to make sure they fully answer the question and explain why their answers are correct. The teacher should encourage the students to use key algebraic terms within their answers such as coefficient, reflection, vertex, maximum, line of symmetry, and roots/zeros/solutions/x-intercepts.

Assessment List and Benchmarks

Students will complete their flipchart, "Calculations Sheet," "Self-Assessment," and "Quadratic Analysis." The self- assessment will also act as the rubric for the teacher when assessing the students' work.

Quadratics Project Steps

1. Open ActivInspire and start a new flipchart by clicking "file," "new" and over to "screen size flipchart"
2. Click "edit" then "grid" then press "visible" in the upper left of the pop up window
3. You will need to pick an origin to create a coordinate plane
4. Pick a starting point (where object is released from)
5. Write the ordered pair of the starting point using a text box ("insert" then "Text"), AND also on your "Calculations Sheet"
6. Pick a 2nd point (it should be relatively close to the starting point)
7. Write the ordered pair ("insert" then "Text"), AND also on your "Calculations Sheet"
8. Pick an ending point (must be about the height of the 2nd point or lower)
9. Write the ordered pair ("insert" then "Text"), AND also on your "Calculations Sheet"
10. Type those 3 ordered pairs in the "STAT" menu on your TI83 or TI84 calculator (stat, edit) Xs in L1 and Ys in L2
11. Press "STAT" again, over to "CALC" and the 5th option is "Quadreg" keep pressing enter until you get your equation/model
12. Write down your equation on your "Calculations Sheet" (round to 3 decimals places)
13. Find the vertex x-coordinate of the vertex $= \dfrac{-b}{2a}$ based on the standard form of a quadratic ($y = ax^2 + bx + c$)
    a. Round to 1 decimal place
14. Plug in the x coordinate back into your equation to get the y coordinate of the vertex (round to 1 decimal place)
15. Plot your vertex and write the ordered pair as well as type it in using a text box
16. Pick any x coordinate between the x of the vertex and the x of the end point.
17. Plug that x back into your equation to get the corresponding y coordinate (round to 1 decimal place)
18. Plot that point and write the ordered pair, as well as inserting a text box

*******Repeat twice more to get a total of 3 paths. Make it colorful and creative. Show all your work on your "Calculations Sheet." *******

Calculations Sheet

Path #1

Starting point

2nd point

Ending point

Equation

Vertex

5th point

Path #2

Starting point

2nd point

Ending point

Equation

Vertex

5th point

Path #3

Starting point

2nd point

Ending point

Equation

Vertex

5th point

Calculations Sheet

Path #1

Starting point $(1,3)$

2nd point $(5,7)$

Ending point $(27,9)$

Equation $y = -.035x^2 + 1.210x + 1.825$

Vertex $x = \dfrac{-b}{2a} = \dfrac{-1.210}{2*(-.035)} = \dfrac{-1.210}{-.07} = 17.3$

$y = -.035(17.3)^2 + 1.210(17.3) + 1.825 = 12.3$ vertex = (17.3 , 12.3)

5th point   plug in x = 22 $y = -.035(22)^2 + 1.210(22) + 1.825 = 11.5$ (22 , 11.5)

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Path #2

Starting point $(1,5)$

2nd point $(5,10)$

Ending point $(29,9)$

Equation $y = -.046x^2 + 1.527x + 3.519$

Vertex $x = \dfrac{-b}{2a} = \dfrac{-1.527}{2*(-.046)} = \dfrac{-1.527}{-.092} = 16.6$

$y = -.046(16.6)^2 + 1.527(16.6) + 3.519 = 16.2$ vertex = (16.6 , 16.2)

5th point   plug in x = 25 $y = -.046(25)^2 + 1.527(25) + 3.519 = 12.9$ (25 , 12.9)

Path #3

Starting point $(1,4)$

2nd point $(4,10)$

Ending point $(27,8)$
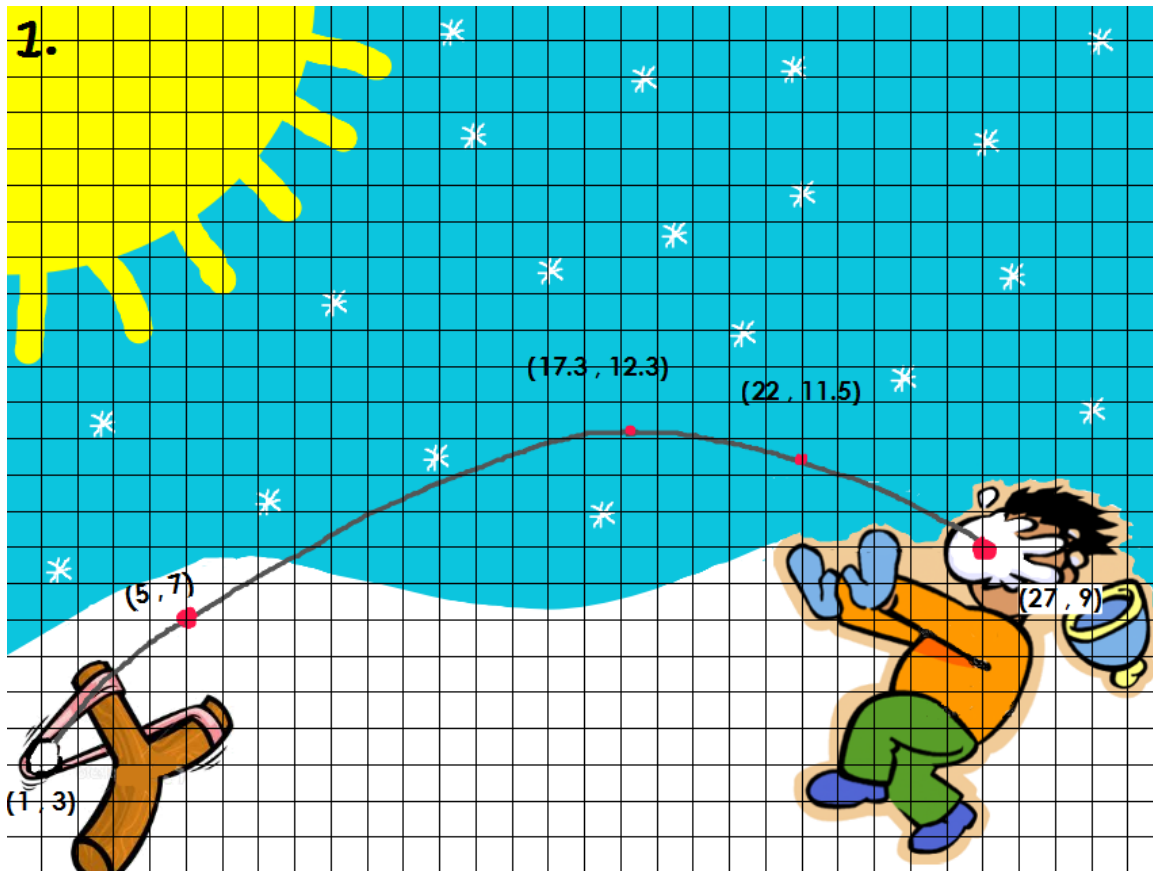
Equation $y = -.080x^2 + 2.401x + 1.679$

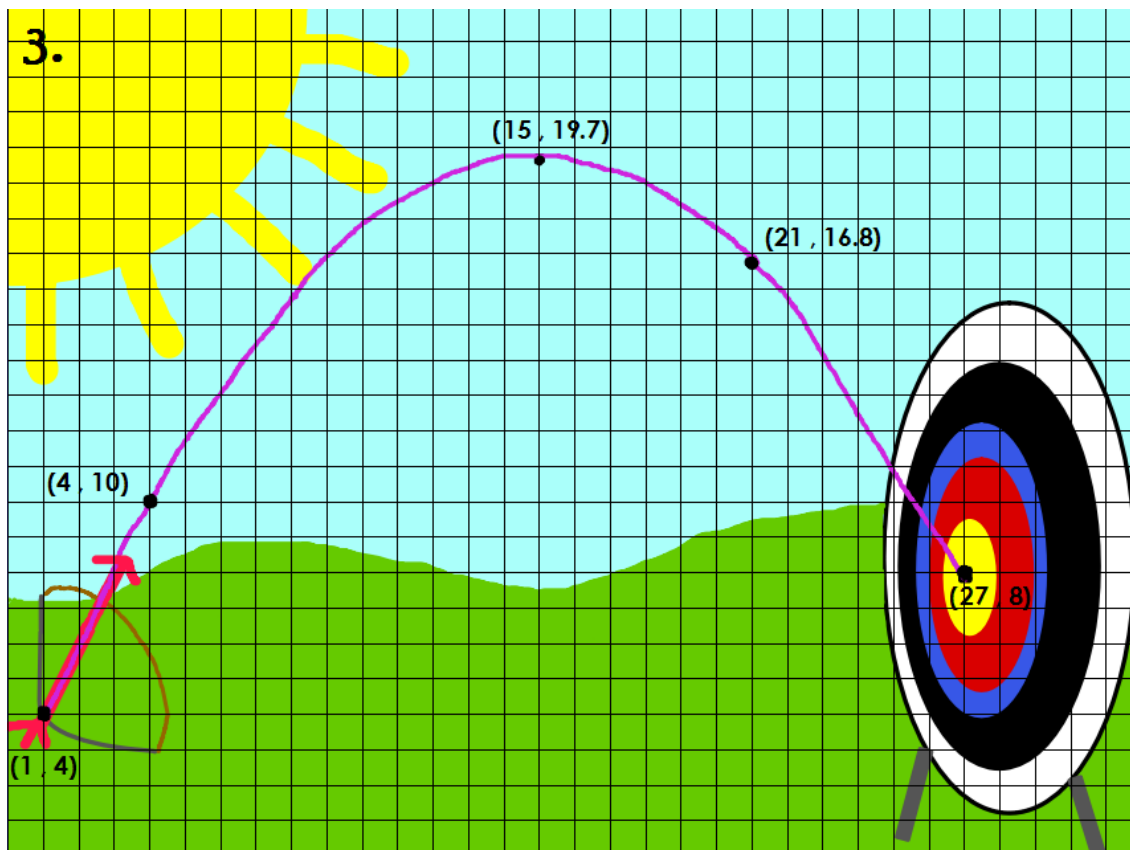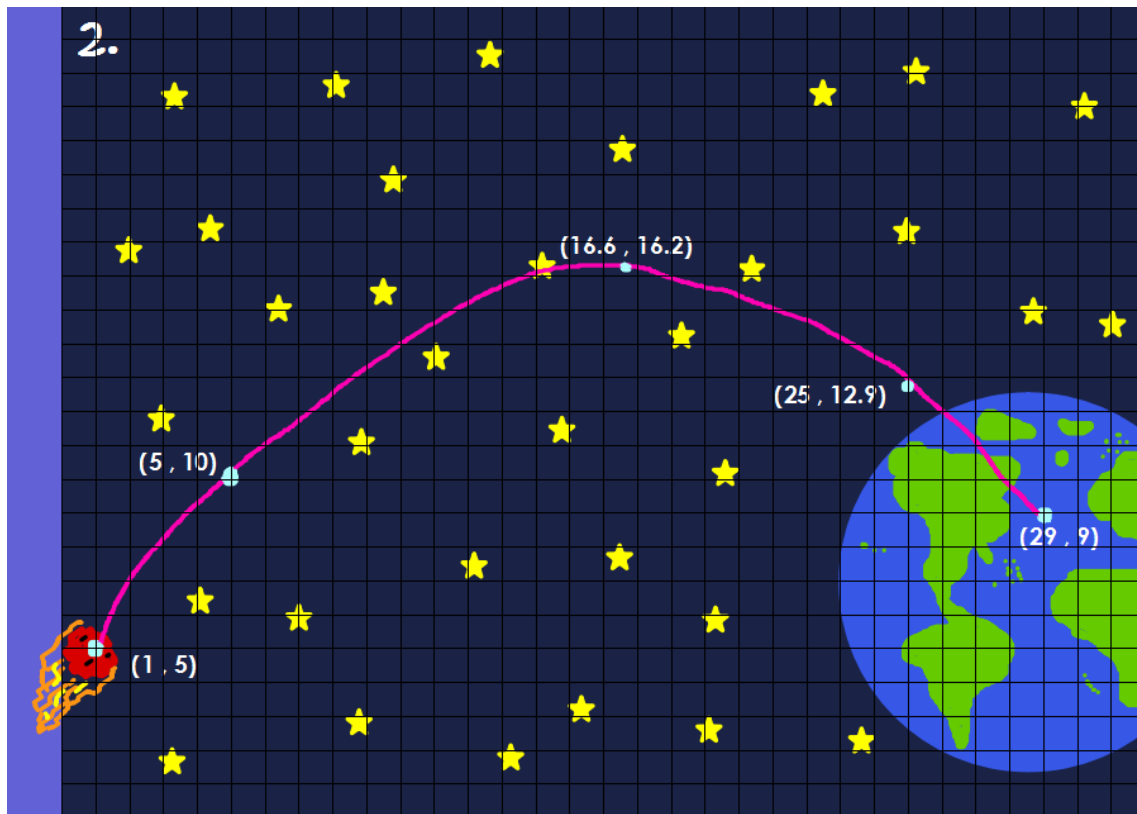Vertex $x = \dfrac{-b}{2a} = \dfrac{-2.401}{2*(-.080)} = \dfrac{-2.401}{-.16} = 15.0$

$y = -.080(15)^2 + 2.401(15) + 1.679 = 19.7$ vertex = (15 , 19.7)

5th point plug in x = 21 $\quad y = -.080(21)^2 + 2.401(21) + 1.679 = 16.8$ (21 , 16.8)

Example of Quadratics Project By Callie Donovan
(Grid kept to help see points for grading)

1.

(17.3 , 12.3)

(22 , 11.5)

(5 , 7)

(1 , 3)

(27 , 9)

2.

(16.6 , 16.2)

(25 , 12.9)

(5 , 10)

(29 , 9)

(1 , 5)

3.

(15 , 19.7)

(21 , 16.8)

(4 , 10)

(27 , 8)

(1 , 4)

Performance Assessment Task – Algebra II   Names_____

"How to Calculate and Analyze the Equation for a Parabolic Path"

Self Assessment- Flipchart   **15 Possible Points**

	3 points	2 points	1 point
Flipchart design and neatness	Flipchart is colored, crisp and neat, and creative	Flipchart is mostly colored, fairly crisp and neat, design is fairly creative	Flipchart isn't colored, and is not original, or neat
Parabolic paths	There are at least 3 paths, paths are smooth and pass through all 5 points	There are at least 2 paths, paths are mostly smooth and pass through at least 4 points	There is at least 1 path, paths aren't very smooth and don't pass through all points
Ordered pairs	Points are marked neatly near the respective point	Points are marked fairly neatly reasonably close to respective point	Points are not marked neatly and/or not close to respective points
Quadratic equations	3 quadratic equations are correctly calculated and in standard form	At least 2 quadratic equations are correctly calculated in standard form	At least 1 quadratic equation is correctly calculated in standard form
Vertex and 5th point	All vertex points and 5th points are calculated correctly and work is shown	At least 4 out of a total of 6 vertex points and 5th points are calculated correctly with work shown	Less than 4 out of a total of 6 vertex points and 5th points are calculated correctly with work shown
Total:			

Performance Assessment Task – Algebra II   Names_____

"How to Calculate and Analyze the Equation for a Parabolic Path"

Teacher Rubric- Flipchart   **15 Possible Points**

	3 points	2 points	1 point
Flipchart design and neatness	Flipchart is colored, crisp and neat, and creative	Flipchart is mostly colored, fairly crisp and neat, design is fairly creative	Flipchart isn't colored, and is not original, or neat
Parabolic paths	There are at least 3 paths, paths are smooth and pass through all 5 points	There are at least 2 paths, paths are mostly smooth and pass through at least 4 points	There is at least 1 path, paths aren't very smooth and don't pass through all points
Ordered pairs	Points are marked neatly near the respective point	Points are marked fairly neatly reasonably close to respective point	Points are not marked neatly and/or not close to respective points
Quadratic equations	3 quadratic equations are correctly calculated and in standard form	At least 2 quadratic equations are correctly calculated in standard form	At least 1 quadratic equation is correctly calculated in standard form
Vertex and 5th point	All vertex points and 5th points are calculated correctly and work is shown	At least 4 out of a total of 6 vertex points and 5th points are calculated correctly with work shown	Less than 4 out of a total of 6 vertex points and 5th points are calculated correctly with work shown
Total:			

Names:_____

__Quadratic Analysis__   (Each question is worth 3 points)

1. Based on the shape of your paths, how could you use your knowledge of quadratics and standard form to predict that "a" would be negative?

2. What is the line of symmetry of your first path? What is the maximum?

3. If your paths represent a function of height with respect to time after release, match x and y with height and time.

4. For your 3rd path, what would a "root" near your ending point represent?

5. If a kid throws a rock off a cliff into the ocean, which is represented by the function $h(t) = -16t^2 + 96t + 112$ where h is height in feet and t is time in seconds

   After how many seconds will the rock reach its maximum height? What is that maximum height? Answer the question by filling in the blanks

   The rock will reach a maximum height of _____ feet after _____ seconds.

6. Use the same scenario and equation that you did for #5…….
   After how many seconds will the rock hit the ocean?

Quadratic Analysis (Each question is worth 3 points)

1. Based on the shape of your paths, how could you use your knowledge of quadratics and standard form to predict that "a" would be negative?

Whenever the quadratic is in the shape of an upside down "U" (reflected downward) the quadratic equation will have a negative number in front of the squared term.

2. What is the line of symmetry of your first path? What is the maximum?

It will be the x-coordinate of the vertex. (x = 17.3) The maximum is the y-coordinate of the vertex (y = 12.3)

3. If your paths represent a function of height with respect to time after release, match x and y with height and time.

X represents time and y represents height

4. For your 3rd path, what would a "root" near your ending point represent?

It represents where the arrow hits the ground

5. If a kid throws a rock off a cliff into the ocean, which is represented by the function $h(t) = -16t^2 + 96t + 112$ where h is height in feet and t is time in seconds

After how many seconds will the rock reach its maximum height? What is that maximum height? Answer the question by filling in the blanks

Will occur at the vertex $t = \dfrac{-96}{2*(-16)} = 3\,\text{seconds}$ $h(3) = -16(3)^2 + 96(3) + 112 = 256$ feet

The rock will reach a maximum height of **256** feet after **3** seconds.

6. Use the same scenario and equation that you did for #5…….
After how many seconds will the rock hit the ocean?

You would find the roots… possibly by factoring $0 = -16\left(t^2 - 6t - 7\right) = -16(t-7)(t+1)$

T = 7   and t = -1   you can't have negative seconds, so the rock would hit the ocean after 7 seconds